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IMAGE ENLARGEMENT BASED ON GREY INTERPOLATION MODEL

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Abstract:

This paper proposes a one-dimensional grey interpolation scheme for image enlargement. This scheme is based on the grey interpolation (GI) model. The GI model is derived through the GM(1,1) model, the implicit GM(1,1) model and the grey data fitting model. In the GI model, a developing coefficient is estimated from available data and applied to find the interpolated value. Several examples are given to justify the proposed scheme. The results indicate that the proposed scheme for image enlargement has similar visual quality to those from popular two-dimensional interpolation schemes, the bilinear interpolation, the bicubic interpolation and the cubic spline interpolation.

Keywords:

Grey model; GM(1,1) model; Grey data fitting model; Grey interpolation model; Image enlargement

1. Introduction

Image enlargement is a scheme to convert a lower resolution image to a higher one. Some applications of image enlargement are videoconference [1], medical imaging [2], and digital photographs [3-4]. Image enlargement can be viewed as an up-sampling problem [5]. Therefore, an interpolation scheme is needed in the up-sampling process. Popular interpolation schemes for image enlargement are the nearest neighbor interpolation (NNI) [6, 8], the bilinear interpolation (BLI) [5-7] and the bicubic interpolation (BCI) [4] and the cubic spline interpolation (CSI) [4]. Those two-dimensional interpolation schemes generally have a satisfactory performance and thus are widely used in many applications where image enlargement is required.

In this paper, a one-dimensional grey interpolation scheme is proposed. Since 1989, many grey models have been developed. The most popular one is the GM(1,1) model which stands for the grey model of first-order and one variable. The schemes related to GM(1,1) model has been applied to many fields, such as short-term load forecasting [12], speech enhancement [13], voice activity detection [14], and so on. However, the ability of GM(1,1) model in data fitting is not good enough. Consequently, there is a difficulty using the GM(1,1) model as a interpolation model. To deal with the problem, the grey data fitting (GDF) model was derived in [11]. Base on the GDF model, a grey interpolation model is introduced and applied to image enlargement. This paper is organized as follows. In Section 2, the related grey models are briefly reviewed. Then the proposed grey interpolation (GI) model is derived from the GDF model. The way to apply GI model to image enlargement is described in Section 3 as well. Then examples are provided to verify the proposed interpolation approach where comparisons with popular two-dimensional interpolation schemes, the NNI, the BLI, the BCI, and the CSI, are given in Section 4. Finally, conclusion is made in Section 5.

2. Review of Grey Models

This section briefly reviews three grey models: the GM(1,1) model, the implicit GM(1,1) model, and the grey data fitting model. These models lay the foundation of the proposed grey interpolation (GI) model which will be described in Section 3 later on. The three models are briefly reviewed in the following.

2.1. The GM(1,1) model

The GM(1,1) model is reviewed here. For details, one may consult [9-10]. Given data $\{x^{(0)}(k), \text{for } 1 \le k \le K\}$, a new data sequence $x^{(1)}(k)$ is found by 1-AGO (first-order accumulated generating operation) as

$$x^{(1)}(k) = \sum_{n=1}^{k} x^{(0)}(n) \tag{1}$$

for $1 \le k \le K$, where the value of $x^{(0)}(k)$ is assumed positive and $x^{(1)}(1) = x^{(0)}(1)$. From (1), it is easy to recover $x^{(0)}(k)$ as

$$x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1) \tag{2}$$

for $2 \le k \le K$. This operation is called 1-IAGO (first-order inverse accumulated generating operation).

By $x^{(0)}(k)$ and $x^{(1)}(k)$, a grey difference equation is formed as

$$x^{(0)}(k)+az^{(1)}(k)=b \tag{3}$$

where

$$z^{(1)}(k)=0.5(x^{(1)}(k)+x^{(1)}(k-1)) \tag{4}$$

for $2\le k\le K$, and parameters a and b are called the developing coefficient and the grey input, respectively. It can be shown that the solution of $x^{(0)}(k)$ is of the following form

$$\hat{x}^{(0)}(k)=(1-e^{a})(x^{(0)}(1)-\frac{b}{a})e^{-a(k-1)} \tag{5}$$

2.2. The implicit GM(1,1) model

An implicit form of GM(1,1) model is derived from (3) and (4) as follows. By (4), (3) can be rewritten as

$$\begin{aligned} &x^{(0)}(k)+az^{(1)}(k)\\ &=x^{(0)}(k)+a[0.5(x^{(1)}(k)+x^{(1)}(k-1))]\\ &=x^{(0)}(k)+a[0.5(x^{(1)}(k)-x^{(1)}(k-1)+2x^{(1)}(k-1))]\\ &=x^{(0)}(k)(1+0.5a)+ax^{(1)}(k-1)=b \end{aligned} \tag{6}$$

where 1-IAGO is applied in the third equality.

From (6), for $k=2$ we have

$$x^{(0)}(2)(1+0.5a)+ax^{(1)}(1)=b \tag{7}$$

and consequently

$$x^{(0)}(2)=\frac{b-ax^{(1)}(1)}{(1+0.5a)}=\frac{b-ax^{(0)}(1)}{(1+0.5a)} \tag{8}$$

In the case of $k=3$, we have

$$\begin{aligned} &x^{(0)}(3)(1+0.5a)+ax^{(1)}(2)\\ &=x^{(0)}(3)(1+0.5a)+a\sum_{n=1}^{2}x^{(0)}(n)\\ &=b \end{aligned} \tag{9}$$

and

$$\begin{aligned} x^{(0)}(3)&=\frac{b-ax^{(0)}(1)-ax^{(0)}(2)}{(1+0.5a)}=\frac{b-ax^{(0)}(1)}{(1+0.5a)}-\frac{ax^{(0)}(2)}{(1+0.5a)}\\ &=x^{(0)}(2)-\frac{ax^{(0)}(2)}{(1+0.5a)}=\frac{1-0.5a}{1+0.5a}x^{(0)}(2) \end{aligned} \tag{10}$$

Similarly, it can be shown

$$x^{(0)}(k+1)=\frac{1-0.5a}{1+0.5a}x^{(0)}(k) \tag{11}$$

for $2\le k\le K-1$. In this implicit GM(1,1) model, (8) and (11) are, respectively, for the cases $k=2$ and $k\ge 3$.

2.3. Grey data fitting model

In this section, we will convert the implicit GM(1,1) model described in 2.2 into an data fitting model called grey data fitting (GDF) model [11], which performs data fitting with excellent accuracy. The derivation of GDF model is given as follows.

Note that the class ratio [9-10] of $\hat{x}^{(0)}(k)$ and $\hat{x}^{(0)}(k-1)$, $\lambda(k)$, is defined as

$$\lambda(k)=\frac{\hat{x}^{(0)}(k)}{\hat{x}^{(0)}(k-1)}=e^{-a} \tag{12}$$

where (5) is used in the second equality. To adaptively calculate developing coefficient from $x^{(0)}(k)$, (12) is modified as

$$\lambda(k)=\frac{x^{(0)}(k)}{x^{(0)}(k-1)}=e^{-a(k)} \tag{13}$$

where $a(k)$ is the kth developing coefficient associated with $x^{(0)}(k)$ and $x^{(0)}(k-1)$. Note that $\hat{x}^{(0)}(k)$ and $\hat{x}^{(0)}(k-1)$ in (12) have been replaced with $x^{(0)}(k)$ and $x^{(0)}(k-1)$ in (13). By taking $\ln(\cdot)$ on both side of (13),

$$a(k+1)=\ln x^{(0)}(k)-\ln x^{(0)}(k+1) \tag{14}$$

Next, (8) and (11) are unified as

$$\hat{x}^{(0)}(k+1)=\frac{1-0.5a(k+1)}{1+0.5a(k+1)}x^{(0)}(k) \tag{15}$$

for $1\le k\le K-1$, where $\hat{x}^{(0)}(1)=x^{(0)}(1)$ is assumed and $a(k+1)$ is calculated as in (14). Note that in (15) $\hat{x}^{(0)}(k+1)$ and $a(k+1)$ have taken the positions of $x^{(0)}(k+1)$ and a in (11), respectively.

In order to improve the fitting performance, the GDF model employs a preprocessing scheme to reduce the range of developing coefficient $a(k)$ and the corresponding post-processing to obtain the estimate of $x^{(0)}(k)$. Since grey 1-AGO is not required in GDF modeling, we drop out the superscript of $x^{(0)}(k)$. Given a data sequence $\{x(k),\ 1\le k\le K\}$ where $x(k)>0$ for all k and K is the total number of data, the modeling procedure of GDF model is described as follows.

Step 1: Obtain a new data sequence $y(k)$ from $x(k)$ as

$$y(k)=\log_{10}x(k) \tag{16}$$

for $1\le k\le K$.

Step 2: Calculate developing coefficient $a(k)$ from $y(k)$ as

$$a(k+1)=\ln y(k)-\ln y(k+1) \tag{17}$$

for $1\le k\le K-1$.

Step 3: Find the estimate of $y(k+1)$, $\hat{y}(k+1)$, as

$$\hat{y}(k+1)=\frac{1-0.5a(k+1)}{1+0.5a(k+1)}y(k) \tag{18}$$

for $1\le k\le K-1$, where $\hat{y}(1)=y(1)$ is assumed.

Step 4: Obtain the estimate $\hat{x}(k)$ from as

$$\hat{x}(k) = 10^{\hat{y}(k)} \tag{19}$$

for $1 \le k \le K$.

In the modeling process of GDF model, the admissible range of $a(k+1)$ is $(-2, 2)$. If $a(k+1)$ is out of the range, the estimate $\hat{x}(k)$ will change its sign from positive to negative. Obviously, the modeling process fails in this case. To deal with the problem, a pixel value less than 8 will be set to 8 when the GDF model is applied to a 256 grey level image. Since the GDF model has almost perfect fitting for image data, therefore a grey interpolation model is developed through the GDF model. The grey interpolation model is described in Section 3.

3. The Proposed Grey Interpolation Model and Image Enlargement

In this section, a grey interpolation (GI) model based on the GDF model is proposed in Section 3.1. Then the GI model is applied to image enlargement which is given in Section 3.2.

3.1. The grey interpolation model

The objective of GI model is to interpolate a value between two adjacent values. Denote two available values as $x(k)$ and $x(k+1)$. The implementation steps of GI model are given as follows.

Step 1. Obtain $y(k)$ from $x(k)$ as

$$y(k) = \log_{10} x(k) \tag{20}$$

for $1 \le k \le 2$.

Step 2. Estimate the developing coefficient $a(k+1/2)$ as

$$\hat{a}(k+1/2) = \alpha \ln y(k) - (1-\alpha)\ln y(k+1) \tag{21}$$

where $0 < \alpha < 1$ is a scaling factor.

Step 3. Find the estimate of $y(k+1/2)$ as

$$\hat{y}(k+1/2) = \frac{1-0.5\hat{a}(k+1/2)}{1+0.5\hat{a}(k+1/2)} y(k) \tag{22}$$

Step 4. Obtain the interpolated value $\hat{x}(k+1/2)$ as

$$\hat{x}(k+1/2) = 10^{\hat{y}(k+1/2)} \tag{23}$$

Note that the proposed GI model is one-dimensional and non-linear. Moreover, only two values are involved in the interpolation. Thus, the computational complexity of GI model is low.

3.2. The GI model based image enlargement

The way to apply the proposed GI model to image enlargement is described here. Assume image $\boldsymbol{O}$ of size $L \times L$ is down-sampled by factor 2×2 . The down-sampled image is denoted as $\boldsymbol{O}_M$. Based on the GI model, the proposed interpolation scheme is given in the following steps.

Step 1. Scan image $\boldsymbol{O}_M$ row by row horizontally. Denote a pair of adjacent pixels as $\{x(k),\ 1 \le k \le 2\}$.

Step 2. If $x(k) < 8$, set $x(k) = 8$.

Step 3. Put $x(1)$ and $x(2)$ into the GI model and obtain the interpolated value $x(1.5)$.

Step 4. Redo Steps 1 to 3 until all interpolated pixels are found.

Step 5. Similarly, find all interpolated pixels on a column by column basis. Then obtain the enlarged image $\hat{\boldsymbol{O}}$.

4. Simulation Results

In this section, six 512×512 images are provided to justify the proposed image enlargement approach based on the GI model. In the simulation, those images are down-sampled to 256×256 . The parameter α in the GI model is set to 0.5 for all cases. To evaluate the performance of the proposed approach, the peak signal-to-noise ratio (PSNR) is employed which is defined as

$$PSNR = 10\log_{10}\frac{255^2}{MSE} \quad \text{(dB)} \tag{24}$$

where

$$MSE = \frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L}[O(i,j) - \hat{O}(i,j)]^2 \tag{25}$$

Notations $O(i,j)$ and $\hat{O}(i,j)$ are elements of $\boldsymbol{O}$ and $\hat{\boldsymbol{O}}$, respectively.

The PSNR of the enlarged images obtained from the proposed approach are shown in Table 1 where GIM denotes the GI model based interpolation scheme. For comparison, the PSNR from the two-dimensional interpolation schemes, the NNI, the BLI, the BCI, and the CSI, are also given in Table 1. From Table 1, the GI model based approach outperforms the NNI while has similar PSNR with the BLI for most of cases. For images Baboon and Goldhill, the proposed approach has better PSNR than those for the BCI and the CSI, even though the proposed GIM is one-dimensional and the BCI and the CSI are two-dimensional.

As for the visual quality, two enlarged images, Peppers and Goldhill, obtained from the proposed approach and the other interpolation schemes are compared. As in Table 1, the PSNR of enlarged Peppers from the GIM is worse than the BLI by about 1 dB and worse than the BCI, the CSI, by about 1.5 dB. Portions of compared enlarged Peppers are given in Figure 1. As shown in Figure 1, the portion of enlarged Peppers by the GIM has similar visual quality to those from the BLI, the BCI, and the CSI. Figure 2 shows portions of enlarged Goldhill for the compared schemes. The portion of enlarged Goldhill obtained from the GIM looks like having a little bit better visual quality. To sum up, the proposed one-dimensional GIM approach generally has approximate visual quality to those two-dimensional schemes.

5. Conclusion

This paper has presented a one-dimensional interpolation scheme based on the GI model. The GI model is derived from the GDF model which is an implicit GM(1,1) model. The proposed approach has been justified by six images where comparisons with the NNI, the BLI, the BCI, the CSI, have been made in terms of PSNR and the visual quality of enlarged image. The simulation results indicated that the proposed approach had similar visual quality of enlarged image to the compared two-dimensional schemes even of less PSNR in many cases. It suggests that the GI model based interpolation scheme is feasible. In further research, the objective is to enhance the interpolation performance of GI model such that the PSNR of enlarged image could be improved.

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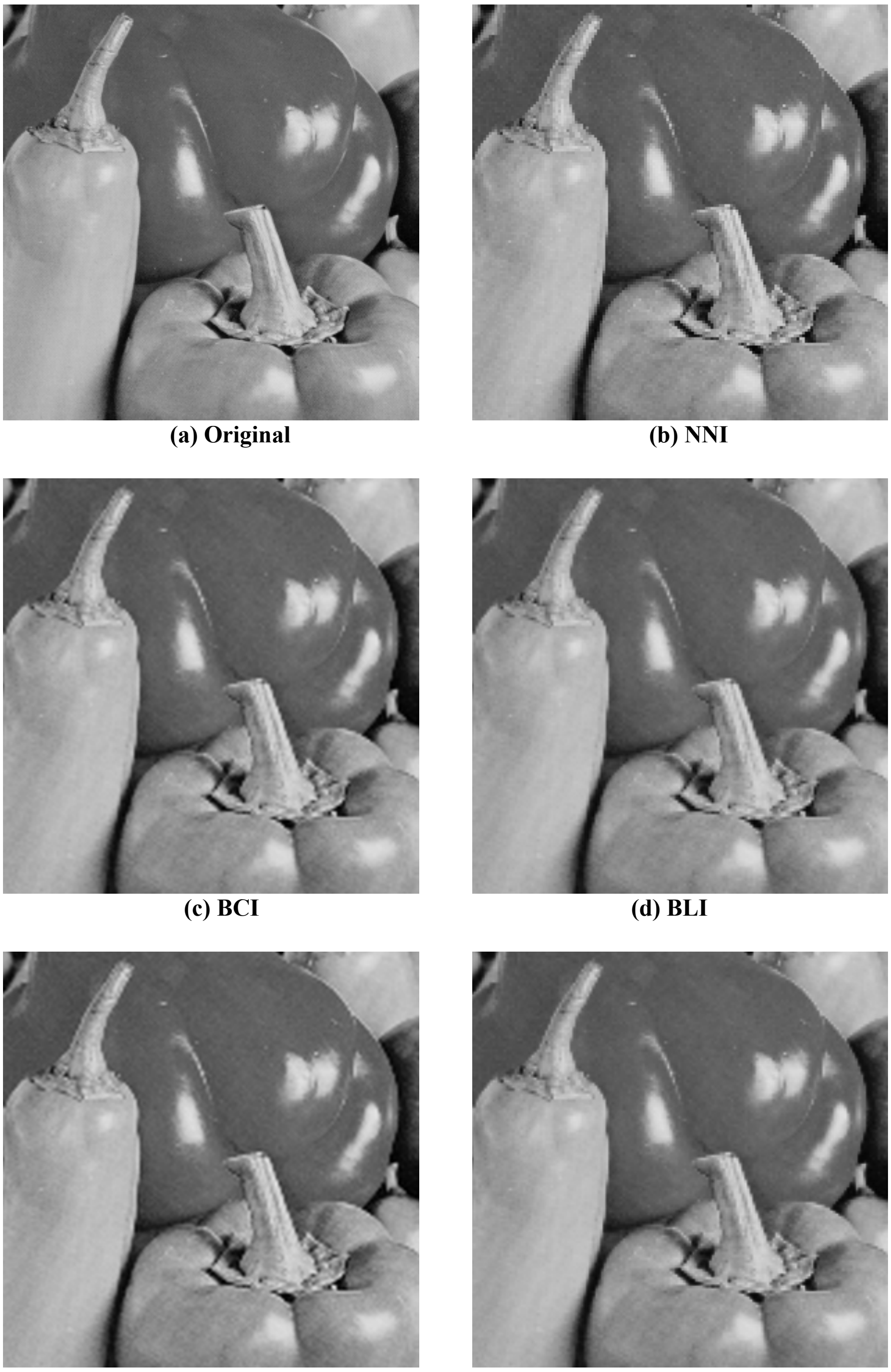
Table 1. The comparison of PSNR for the proposed approach and other schemes

	Lena	Peppers	Baboon	Jet	Lake	Goldhill
NNI	28.34	28.27	20.41	26.55	25.32	27.23
BLI	33.46	31.48	22.68	30.82	29.10	30.70
BCI	34.00	32.04	22.47	31.57	29.55	30.59
CSI	33.97	32.14	22.16	31.76	29.54	30.28
GIM	33.28	30.50	22.59	30.56	28.71	30.63

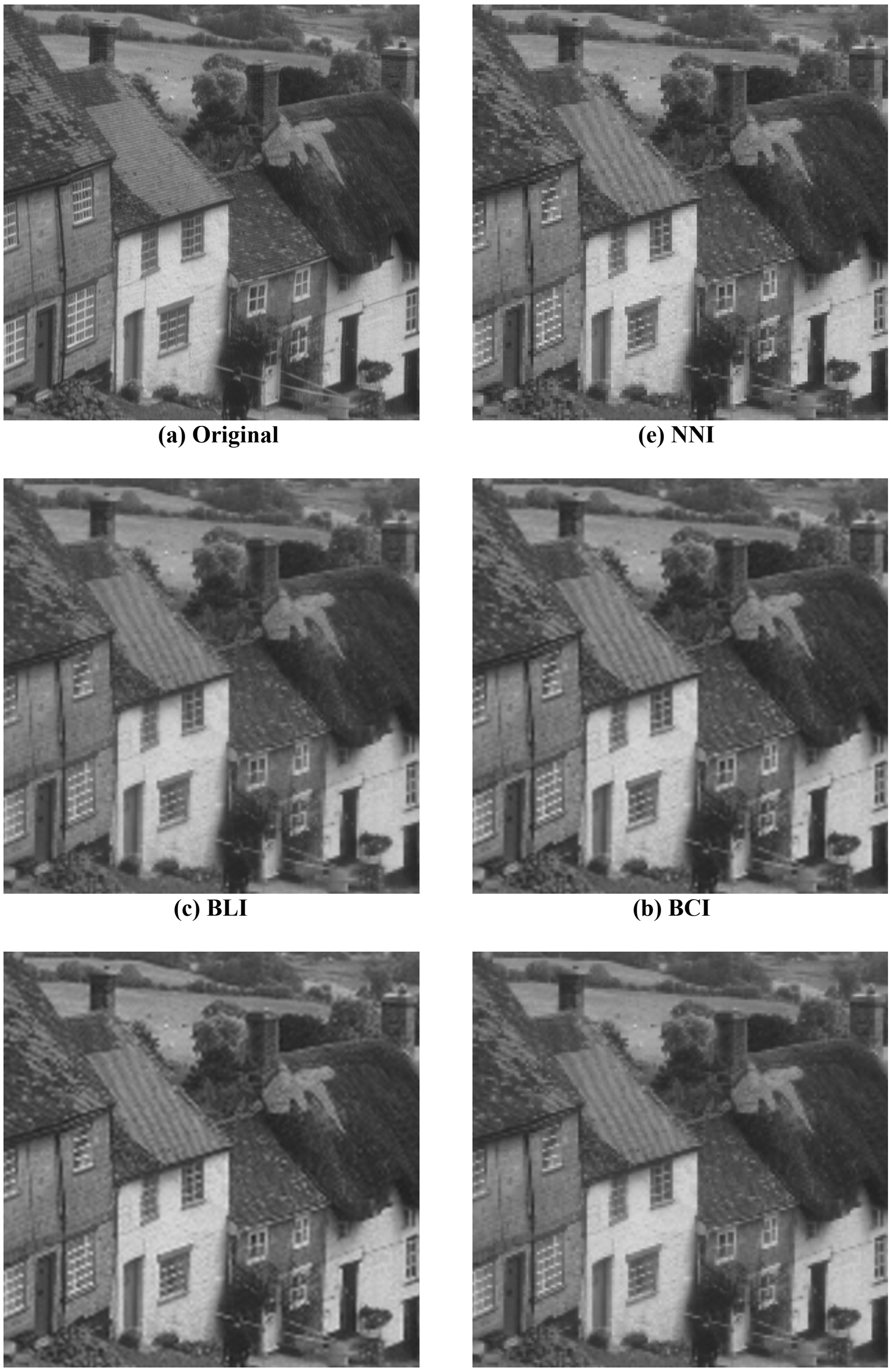


(a) Original **(b) NNI**

(c) BCI **(d) BLI**

(e) CSI **(f) GIM**

Figure 1. The enlarged Peppers obtained from the GIM and other schemes



(a) Original **(e) NNI**

(c) BLI **(b) BCI**

(d) CSI **(f) GIM**

Figure 2. The enlarged Peppers obtained from the GIM and other schemes